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LIGHT-CONE PARAMETRIZATIONS FOR KÄHLER MANIFOLDS

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Abstract

It is shown that, for any Kähler manifold, there exist parametrizations such that the metric takes a block-form identical to the light-cone metric introduced by Polyakov for two-dimensional gravity. Besides its possible relevence for various aspects of Kählerian geometry, this fact allows us to change gauge in W gravities, and explicitly go from the conformal (Toda) gauge to the light-cone gauge using the W-geometry we proposed earlier (this will be discussed in detail in a forthcoming article).

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It is hardly necessary to stress the importance of Kähler manifolds. They arise in various important problems of theoretical physics and mathematics. For instance, Kähler geometries play an important role in supergravity theories and string compactification[1][2]. For us, the motivation to study Kähler manifolds is our recent works[3][4], where they were connected with W-geometries. In particular, it was shown in ref.[4] that the classical solutions of A_n-W gravity in the conformal gauge, are in a one-to-one correspondence with holomorphic surfaces in the complex projective space CP^n, which is the standard non-trivial example of Kähler manifolds. On the other hand, W gravities have also been studied[5][6][7] in a light-cone approach[2] similar to the one originally introduced for two-dimensional gravity[8]. So far these two gauges had not been connected. In our study of W-geometries, we came across a general result about Kähler manifolds which is the point of this letter: starting from any set of coordinates of the usual type, there exist reparametrizations such that the metric takes a block-form identical to the light-cone metric of two-dimensional gravity introduced in ref.[8]. This results allows us to connect light-cone and conformal descriptions of W gravities, as we will show in details in a forthcoming paper. It is, however, of a more general interest, and we present it separately in this letter.

We shall deal with an arbitrary Kähler manifold $\mathcal{M}$ of real dimension $2n$. By definition there exists a special class of coordinates X^A, $X^{\bar A}$, $1 \leq A$, $\bar A \leq n$, such that the only components of the metric are $G_{A\bar B} = G_{\bar B A}$, and

$$G_{A\bar B} = \partial_A \partial_{\bar B} K, \tag{1}$$

where K is the Kähler potential. In general we denote the differential operators $\partial/\partial X^A$, and $\partial/\partial X^{\bar B}$ by ∂_A, and $\partial_{\bar B}$. These coordinates will be called conformal since they appear in the geometrical description of W gravity in the conformal gauge. They will be collectively denoted as $X^{\underline{A}}$, with $1 \leq \underline{A} \leq 2n$. Our basic point is the definition of another set of prefered coordinates denoted $U^{\underline{A}}$ such that the new metric tensor denoted $H_{\underline{A}\,\underline{B}}$ takes a form which is standard in the light-cone approach to W-gravity. The $U^{\underline{A}}$ will be called light-cone coordinates. In the same way as the conformal coordinates, they are split into two sets denoted U^A, and $U^{\bar A}$, respectively. The change of

[2]it is also called chiral gauge, but for us this terminology would lead to confusion, so that we do not use it.

coordinates is of the form

$$U^{\bar{A}} = U^{\bar{A}}(X^1, \cdots, X^n; X^{\bar{1}} \cdots, X^{\bar{n}}), \quad U^A = X^A. \tag{2}$$

The functions $U^{\bar{A}}(X;\bar{X})$ will be determined next, in such a way that the light-cone metric takes the form

$$H_{\bar{A}\,\bar{B}} = 0, \quad H_{A\,B} = 2h_{A\,B}$$
$$H_{A\,\bar{B}} = H_{\bar{B}\,A} = \delta_{A,\,\bar{B}}, \tag{3}$$

where $h_{A\,B}$ will be related to the Kähler potential. Before going on, let us remark that the determinant of $H_{\underline{AB}}$ is equal to -1, and that its inverse $H^{\underline{A}\,\underline{B}}$ is given by

$$H^{A\,B} = 0, \quad H^{\bar{A}\,\bar{B}} = -2h_{A\,B}$$
$$H^{A\,\bar{B}} = H^{\bar{B}\,A} = \delta_{A,\,\bar{B}}. \tag{4}$$

On the contrary, no general form may be given for the determinant of G or its inverse. This is a very nice point of the light-cone parametrization[3]. Going back to our main line, one finds, by standard computations, that conditions Eqs.3 are fulfilled if one has

$$G_{A\,\bar{B}} = H_{A\,\bar{C}}\partial_{\bar{B}}U^{\bar{C}} \tag{5}$$

$$2h_{A\,B} = -H_{B\,\bar{C}}\partial_A U^{\bar{C}} - H_{A\,\bar{C}}\partial_B U^{\bar{C}} \tag{6}$$

These equations are easily solved by using the Kähler potential, obtaining

$$U^{\bar{A}} = H^{\bar{A}\,C}\partial_C K, \tag{7}$$

$$h_{A\,B} = -\partial_A\partial_B K. \tag{8}$$

Thus we reach the important conclusion that, for any Kählerian manifold there is a choice of coordinates such that the metric tensor takes the block-form

$$H = \begin{pmatrix} 2h & 1 \\ 1 & 0 \end{pmatrix} \tag{9}$$

which is the same as the one introduced by Polyakov to describe 2D gravity in the light-cone gauge.

[3] inverting the metric tensor is often a pain in the neck.

In the same way as for two-dimensional gravity in the light-cone gauge, it is convenient to develop the tensor calculus appropriate to this light-cone parametrization. For this, it is useful to introduce once for all a special notation. In many cases, we have to contract indices, not with the full metric tensor H, but only with its off-diagonal part $H_{A\,\bar B}=\delta_{A\,\bar B}$ or with $H^{A\,\bar B}=\delta_{A\,\bar B}$. In practice this leads to equate numerical values of bar and unbar indices leading to confusions between bar and unbar components. We shall use the following convention. Consider a covariant vector $V^{\underline{A}}$. The index is raised and lowered using the usual convention: $V_{\underline{B}}=H_{\underline{B}\,\underline{A}}V^{\underline{A}}$, and so on. However, it is convenient to define

$$
\begin{aligned}
V^{\overline{\widehat{A}}} &\equiv H_{A\,\bar B}V^{\bar B}, \quad V^{\widehat{\overline{A}}}\equiv H_{\bar A\,B}V^{B};\\
V_{\overline{\widehat{A}}} &\equiv H^{A\,\bar B}V_{\bar B}, \quad V_{\widehat{\overline{A}}}\equiv H^{\bar A\,B}V_{B}.
\end{aligned} \tag{10}
$$

The basic idea of this notation is that this contraction sets numerical values of indices equal but does not change their tensorial character. Thus, for instance, the left equation of the first line above contains the upper bar-component of V with the numerical value of the index set equal to A. As an example of this rule, the correspondence between covariant and contravariant vectors explicitly reads

$$
\begin{aligned}
&V_A=V^{\overline{\widehat{A}}}+2h_{AB}V^B, \quad V_{\bar A}=V^{\widehat{\overline{A}}}\\
&V^{\bar A}=V_{\widehat{\overline{A}}}-2h_{\widehat{\overline{A}}\widehat{\overline{B}}}V_{\bar B}, \quad V^A=V_{\overline{\widehat{A}}}.
\end{aligned} \tag{11}
$$

Another example is that Eq.7 becomes

$$
U^{\overline{\widehat{A}}}=\partial_A K \tag{12}
$$

Concerning the Christoffel symbols denoted Γ, the usual text-book calculation gives in the present case

$$
\begin{aligned}
\Gamma^{C}_{AB} &= -D_{\overline{\widehat{C}}}h_{AB}\\
\Gamma^{\bar C}_{AB} &= D_A h_{\widehat{\overline{C}}B}+D_B h_{A\widehat{\overline{C}}}-D_{\widehat{\overline{C}}}h_{AB}+2h_{\widehat{\overline{C}}\widehat{\overline{M}}}D_{\bar M}h^{AB}\\
\Gamma^{C}_{A\,\bar B} &= D_{\bar B}h_{A\widehat{\overline{C}}}\\
\Gamma^{C}_{A\bar B} &= \Gamma^{C}_{\bar A\bar B}=\Gamma^{\bar C}_{\bar A\bar B}=0.
\end{aligned} \tag{13}
$$

The partial derivatives with respect to U^A and $U^{\bar{A}}$ are denoted by D_A and $D_{\bar{A}}$.

Our next topic is to look for coordinate reparametrizations that leave the light-cone form Eq.3 invariant. Under the infinitesimal change of coordinate $U^{\underline{A}} = \widetilde{U}^{\underline{A}} + \epsilon^{\underline{A}}$, the variation of the metric is

$$\delta H_{\underline{A}\,\underline{B}} = \mathcal{D}_{\underline{A}} \epsilon_{\underline{B}} + \mathcal{D}_{\underline{B}} \epsilon_{\underline{A}}$$

where $\mathcal{D}$ denotes covariant derivatives with respect to the Christoffel symbols Eqs.13. Imposing first that $H_{A\,\bar{B}}$ and $H_{\bar{A}\,\bar{B}}$ remain unchanged gives

$$\delta H_{A\,\bar{B}} = 0 = D_A \epsilon_{\bar{B}} + D_{\bar{B}} \epsilon_A - 2(D_{\bar{B}} h_{A\,\overline{\widehat{C}}})\, \epsilon_{\bar{C}} \tag{14}$$

$$\delta H_{\bar{A}\,\bar{B}} = 0 = D_{\bar{A}} \epsilon_{\bar{B}} + D_{\bar{B}} \epsilon_{\bar{A}} \tag{15}$$

Assuming that these conditions hold, we consider the variation of H_{AB}, that is

$$\delta H_{AB} = D_A \epsilon_B + D_B \epsilon_A - 2\Gamma^C_{AB} \epsilon_C - 2\Gamma^{\bar{C}}_{AB} \epsilon_{\bar{C}}$$

Next we show that, in close analogy with 2D gravity, this may be rewritten solely as a function of $\epsilon^{\overline{\widehat{C}}} + h_{CL}\epsilon^L$. This is achieved as follows: using Eq.11 we write

$$\delta H_{AB} = \Big[D_A \delta_{CB} + D_B \delta_{CA} - 2\Gamma^C_{AB} \Big] (\epsilon^{\overline{\widehat{C}}} + 2h_{CL}\epsilon^L) - 2\Gamma^{\bar{C}}_{AB} \epsilon_{\bar{C}}.$$

In the first term, we change the coefficient of h_{CL} from 2 to 1, and introduce $\tilde{\delta} H_{AB}$ such that

$$\delta H_{AB} = \Big[D_A \delta_{CB} + D_B \delta_{CA} - 2\Gamma^C_{AB} \Big] (\epsilon^{\overline{\widehat{C}}} + h_{CL}\epsilon^L) + \tilde{\delta} H_{AB}.$$

$$\tilde{\delta} H_{AB} = D_A(h_{BL}\epsilon^L) + D_B(h_{AL}\epsilon^L) - 2\Gamma^C_{AB} h_{CL}\epsilon^L - 2\Gamma^{\bar{C}}_{AB} \epsilon_{\bar{C}}$$

Next, we make use of Eqs.14, and 15, and derive

$$(h_{BC} D_A + h_{AC} D_B)\epsilon^C =$$

$$-h_{BC} D_{\overline{\widehat{C}}} \Big[\epsilon^{\overline{\widehat{A}}} + h_{AL}\epsilon^L \Big] - h_{AC} D_{\overline{\widehat{C}}} \Big[\epsilon^{\overline{\widehat{B}}} + h_{BL}\epsilon^L \Big]$$

$$+ h_{BC} D_{\overline{\widehat{C}}}(h_{AL})\epsilon^L + h_{AC} D_{\overline{\widehat{C}}}(h_{BL})\epsilon^L.$$

$$
\tilde{\delta} H_{AB} = -\left[h_{CB}\delta_{AL}D_{\overline{\widehat{C}}} + h_{CA}\delta_{BL}D_{\overline{\widehat{C}}}\right](\epsilon^{\overline{\widehat{C}}} + h_{CL}\epsilon^{L}) + V_C\,\epsilon^{C}
$$

$$
V_C = h_{BL}D_{\overline{\widehat{L}}}h_{AC} + h_{AL}D_{\overline{\widehat{L}}}h_{BC} + D_A h_{BC} + D_B h_{AC} - 2\Gamma^{L}_{AB}h_{LC} - 2\Gamma^{\overline{\widehat{C}}}_{AB}.
$$

Finally, one verifies that V_C is actually equal to zero. This is a consequence of the Kähler condition, which allows us to transform the expression of $\Gamma^{\overline{\widehat{C}}}_{AB}$. Indeed, using the fact that $\partial_A = D_A - h_{AL}D_{\overline{\widehat{L}}}$, one sees that the expression for $\Gamma^{\overline{\widehat{C}}}_{AB}$ given in Eq.13 is equivalent to

$$
\Gamma^{\overline{\widehat{C}}}_{AB} = \frac{1}{2}\Big(D_A h_{BC} + D_B h_{AC} + h_{AL}D_{\overline{\widehat{L}}}h_{BC} + h_{BL}D_{\overline{\widehat{L}}}h_{AC}\Big) + h_{CL}D_{\overline{\widehat{L}}}h_{AB}. \tag{16}
$$

Substituting this last expression into the above formula for V_C one finds that there is a complete cancellation between the last term and the others. Collecting the remaining pieces, one arrives at the formula

$$
\delta H_{AB} =
$$

$$
\left[D_A\delta_{CB} + D_B\delta_{CA} - (h_{B\widehat{\bar{M}}}\delta_{AC} + h_{A\widehat{\bar{M}}}\delta_{BC})D_{\bar{M}} + 2D_{\overline{\widehat{C}}}h_{AB}\right](\epsilon^{\overline{\widehat{C}}} + h_{CL}\epsilon^{L}). \tag{17}
$$

As already announced, it only involves the quantities

$$
v_C \equiv \epsilon^{\overline{\widehat{C}}} + h_{CL}\epsilon^{L} = \epsilon_C - h_{CL}\epsilon^{L}. \tag{18}
$$

It may be written compactly as

$$
\delta H_{AB} = (\nabla_A v)_B + (\nabla_B v)_A \tag{19}
$$

$$
(\nabla_A v)_B \equiv (D_A - h_{A\widehat{\bar{M}}}D_{\bar{M}})v_B + (D_{\overline{\widehat{C}}}h_{AB})\,v_C. \tag{20}
$$

This generalizes a basic formula of 2D gravity in the light-cone gauge which reads, with standard notations, $\delta h = (\partial_+ - h\partial_- + \partial_+ h)(\epsilon^+ + h\epsilon^-)$.

The usual Kähler formulation is obviously covariant under holomorphic change of coordinates $X^A \to X^A(\widetilde{X}^1, \cdots, \widetilde{X}^n)$, $X^{\bar{A}} \to X^{\bar{A}}(\widetilde{X}^{\bar{1}}, \cdots, \widetilde{X}^{\bar{n}})$. Then $K \to \widetilde{K}$ such that $K(X, \bar{X}) = \widetilde{K}(\widetilde{X}, \widetilde{\bar{X}})$. Using the change to light-cone coordinates just displayed, this gives examples of transformations that leave the light-cone form invariant. Consider the infinitesimal holomorphic transformation

$$
X^A = \widetilde{X}^A - \eta^A(X), \quad X^{\bar{A}} = \widetilde{X}^{\bar{A}} - \eta^{\bar{A}}(\bar{X}). \tag{21}
$$

It immediately follows from Eq.12 that

$$\delta U^{\overline{\bar{A}}} = -(\partial_A \eta^C)\partial_C K = -U^{\overline{\bar{C}}} D_A \eta^C, \tag{22}$$

and Eq.2 directly shows that

$$\delta U^A = \eta^A. \tag{23}$$

Thus the antiholomorphic part $\eta^{\bar{A}}(\bar{X})$ does not act, and the variation of the $\bar{U}$ coordinates is linear in $\bar{U}$. One sees that, when one goes to the light-cone coordinates, the covariance under anti-holomorphic transformations is lost. On the other hand, other transformations appear. First, the light-cone formulation is not invariant under the change of the Kähler potential

$$\delta K = \phi(X) + \bar{\phi}(\bar{X}) \tag{24}$$

that leave the original metric Eq.1 invariant. A simple calculation shows that

$$\delta U^{\overline{\bar{A}}} = -D_A \phi, \quad \delta U^A = 0 \tag{25}$$

This, give another example of transformations of the type Eq.20. Second, we did not write down the most general solution of Eqs.5, and 6. It is given by

$$U^{\overline{\bar{A}}} = \partial_A K + \Omega_A(X), \quad h_{AB} = -\partial_A \partial_B K - \frac{1}{2}(\partial_A \Omega_B(X) + \partial_B \Omega_A(X)). \tag{26}$$

Ω_A are arbitrary functions of $X^1, \cdots, X^n$, or, equivalently, of $U^1, \cdots, U^n$. Changing Ω gives another set of transformations that leave the physics invariant, and give examples of Eq.19, and 20.

The change of coordinates just described came out in our study of W gravity as follows. As already recalled, we showed in ref.[4], that the A_n-W–geometry corresponds to the embedding of holomorphic two-dimensional surfaces in CP^n. These (W) surfaces are specified by embedding equations of the from $X^A = f^A(z)$, $\bar{X}^{\bar{A}} = \bar{f}^{\bar{A}}(\bar{z})$, where z, and $\bar{z}$ are the two surface-parameters. The fact that they are functions of a single variable is equivalent to the Toda field-equations, so that this describes W gravity in the conformal gauge. These functions have a natural extension to CP^n using the higher variables $z^{(k)}$, $\bar{z}^{(k)}$ of the Toda hierarchy of integrable flows, and this provides a local parametrization of CP^n. The original variables z and $\bar{z}$ are identified with $z^{(1)}$, $\bar{z}^{(1)}$, respectively. For the embedding functions the extension is such that they become functions of half of the variables noted $f^A([z]) = f^A(z^{(0)}, \cdots z^{(n)})$, and $\bar{f}^{\bar{A}}([\bar{z}]) = \bar{f}^{\bar{A}}(\bar{z}^{(0)}, \cdots \bar{z}^{(n)})$ such that

$$\frac{\partial f^A([z])}{\partial z^{(k)}} = \frac{\partial^k f^A([z])}{(\partial z)^k}, \qquad \frac{\partial \bar{f}^{\bar{A}}([\bar{z}])}{\partial \bar{z}^{(k)}} = \frac{\partial^k \bar{f}^{\bar{A}}([\bar{z}])}{(\partial \bar{z})^k} \tag{27}$$

One main virtue of the coordinates $z^{(k)}$, $\bar{z}^{(k)}$ is that, due to the last equations, higher derivaties in z and $\bar{z}$ are changed to first-order ones, and this is how our geometrical scheme gets rid of the troublesome higher derivatives of the usual approachs. So far this is only for the conformal gauge. Our new result is that, **performing the change of coordinates Eqs.2, 7, 8 in the target space CP^n, allows us to go from W-gravity in the conformal gauge to W-gravity in the light-cone gauge**. From this viewpoint, the transformations Eqs.19 – 21 just display the local gauge group of W gravity in the light-cone gauge. They only involve first order derivatives in the target space. Higher derivatives appear when the higher coordinates are eliminated by means of Eq.27. This will be spelled out later on in full details. Right now we discuss another general aspect, inspired by the problem of W gravity, which is the existence of a Lax pair in the conformal gauge, with a vector-potential related with h_{AB}. We denote by small gamma's the Christoffel symbols in the conformal parametrization. As is well known, the only non-zero components are γ^C_{AB}, and $\gamma^{\bar{C}}_{\bar{A}\bar{B}}$, so that we have

$$\partial_M G_{\bar{A}B} = G_{\bar{A}C}\gamma^C_{MB}, \qquad \partial_{\bar{M}} G_{\bar{A}B} = \gamma^{\bar{C}}_{\bar{M}\bar{A}} G_{\bar{C}B}.$$

This may be easily rewritten in a Lax-pair from

$$\partial_M G_{\bar{A}C} = \mathcal{A}^{\bar{B}}_{(M)\bar{A}} G_{\bar{B}C}, \qquad \partial_{\bar{M}} G_{\bar{A}C} = \mathcal{A}^{\bar{B}}_{(\bar{M})\bar{A}} G_{\bar{B}C}, \tag{28}$$

where

$$\mathcal{A}^{\bar{B}}_{(M)\bar{A}} = G_{\bar{A}C}\gamma^C_{MD} G^{D\bar{B}}, \qquad \mathcal{A}^{\bar{B}}_{(\bar{M})\bar{A}} = \gamma^{\bar{B}}_{\bar{M}\bar{A}}. \tag{29}$$

Moreover, it follows from the Kähler condition Eq.1, that

$$\mathcal{A}^{\bar{B}}_{(M)\bar{A}} = -(\partial_{\bar{A}} H_{MD}) G^{D\bar{B}}, \tag{30}$$

and this establishes the connection between the present vector potential and the light-cone metric tensor Eq.3. It is straightforward to verify that the connection $\mathcal{A}$ is indeed flat, so that the last equation does define a Lax pair. What is the meaning for W gravity ? Consider the components $\mathcal{A}_{(1)}$, and $\mathcal{A}_{(\bar{1})}$. We showed in ref.[4] that when one uses the $z^{(k)}$, and $\bar{z}^{(k)}$, as homogeneous coordinates for CP^n, the Christoffel symbols become very simple. The connection $\mathcal{A}_{(\bar{1})}$ is given by

$$\mathcal{A}_{(\bar{1})} = I + \bar{\lambda} \tag{31}$$

$$I = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & \cdots & & 0 & 1 \\ 0 & \cdots & & 0 & 0 \end{pmatrix}, \quad \bar{\lambda} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & & \cdots & 0 & 0 \\ \bar{\lambda}_0 & \bar{\lambda}_1 & \cdots & \bar{\lambda}_{n-1} & \bar{\lambda}_n \end{pmatrix}, \tag{32}$$

where the $\bar{\lambda}_i$ are related with the W charges. So far we were considering points in the target space CP^n. According to our previous work[4], the usual two dimensional dynamics of W gravity is recovered if one returns to the W surface by letting $z^{(k)} = 0$, and $\bar{z}^{(k)} = 0$, for $k \neq 1$, and $z^{(1)} = z$, and $\bar{z}^{(1)} = \bar{z}$. Then the form of $\mathcal{A}_{(\bar{1})}$ is precisely the one that comes out in the Drinfeld-Sokolov equation[9], in the Hamiltonian reduction[10], and in the generalized Beltrami-differential approach[11]. In particular, this last reference displays a connection between the Lax pair just written, and W gravity in the light-cone gauge. What we just described gives the geometrical origin of this connection, which basically follows from change of coordinates in the target space CP^n. In refs.[7], and [11], it is shown that the anomaly equations of light-cone W gravity precisely comes from the zero-curvature conditions associated with a Lax pair of the type we just wrote. In our approach they thus follow from the Kähler condition together with the existence of a parametrization where the Christoffel symbols take the form Eq.32. There is a subtle difference between the two, however, since starting from Toda field equations we can only get conformally invariant results, contrary to the work of ref.[11]. This point will be discussed in detail later on.

The condition on the Christoffel symbols is strongly reminiscent of the one that specifies a particular Toda theory, in the group-algebraic approach[12] – [14] to Toda dynamics, and in the conformally reduced WZNW approach[15]. Thus a similar mechanism will probably work for the other W geometries.

In conclusion, we have displayed a change of coordinates for arbitrary Kähler geometries that leads to a metric tensor similar to the one of the light-cone formulation of W gravities. These geometries are so important in various problems, that this result will probably be usefull beyond the W gravity problems which was the motivation of this work.

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